Generalized DQ Model of the Permanent Magnet Synchronous Motor Based on Extended Park Transformation

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***Abstract*—A generalized dq model of the permanent magnet synchronous motor (PMSM) based on newly defined extended Park transformation is presented. The new model can describes all kinds of editions of the PMSM dq model together, such as the edition based on Park transformation, the edition based on the principle of power invariance, and so on, for which it can avoid confusion and prevent misusing of different editions at root. The model equations derivation is shown step by step and model compatibility is also verified. The simulation of the vector controlled PMSM drive based on the new model, which is instantiated to some edition, is performed to investigate the performance of the drive in Matlab. The simulation results confirm the new model's compatibility further.**

I. INTRODUCTION

The permanent magnet synchronous motor (PMSM) has been gradually taking the place of other kinds of motors in drive applications such as adjustable speed drives, position servo, and robotics. There are various reasons for the variation tendency. For example, unlike the DC motor and conventional wound-rotor synchronous motor (SM), the PMSM doesn't need the brushes and slip rings which imply safety problem of sparks, regular maintenance and downtime[1]. Moreover, the PMSM has lots of advantages, such as high efficiency, high power density, large torque to inertia ratio, and so on. However, there exist problems such as inherent coupling and nonlinear characteristics in the PMSM. Hence, vector control is usually employed to control it and obtain high dynamic performance which was previously only achievable by the DC motor. Vector control was first presented by F. Blaschke[2], whose original purpose was to improve the performance of the inductor motor (IM). After that, vector control attracted a considerable amount of researchers' interest. Then, people gradually realized the fact that the vector control can be applied to other AC motors such as the PMSM[1]. More than that, it is easier to implement the vector control on the PMSM, because the IM has slip frequency current and more sensitive parameter variations.

The model of the PMSM is the very basis of performing the vector control. The operation principles of the PMSM and of the SM are very similar. Therefore, the dq model of the PMSM can be derived from the well-known dq model[3] of the SM which is based on the dq0 transformation theory. The dq0 transformation first presented by R. H. Park[4] in the late 1920s is widely considered as a sharp revolution of electric machines analysis, so dq0 transformation usually means the Park transformation. However, it's proved that Park transformation is changed to be an orthogonal transformation when it comes to the principle of power invariance[5]. Accordingly, the PMSM has the most widely used two editions of dq model based on these two editions of dq0 transformation.

What is less well known is the fact that the PMSM also has other editions of dq model when the Park transformation is changed to be other editions of dq0 transformation. Because of the similarity and complication of all editions of the dq0 transformation and the PMSM dq model, some confusion among them sometimes exists both in the industry and in the university research environment. So the purpose of this paper is to present a generalized dq model (GDQ model) of the PMSM based on newly defined extended Park (EPark) transformation, which can describe almost all editions of the PMSM dq model derived from different editions of dq0 transformation in detail together. In other words, the GDQ model of the PMSM can describe both the commonality and the difference among almost all editions of the PMSM dq model, for which it can avoid confusion and prevent misusing of them at root.

In this paper, the detailed derivation of the GDQ model and its compatibility verification are presented. Furthermore, vector control of the PMSM based on the GDQ model instantiated to some edition is implemented in Matlab. The inverter used to drive the PMSM is operated based on space

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vector pulse width modulation technique. For current and speed controllers proportional and integral controller is used.

II. EPARK TRANSFORMATION

As is well known, dq0 transformation of the synchronous machine equations from the abc frame to the dq frame can make all sinusoidally varying inductances in the abc frame become constant in the dq frame, from which the dq model based on vector control in PMSM drives has been used to verified high performance[6]. The dq0 transformation that meets projection operation is also called Park transformation, written as equation (1) and represented in a vector diagram shown in Fig. 1.

$$T_p = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta-4\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta-4\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \quad (1)$$

Define the vectors in the fame abc, $\alpha\beta$ and dq respectively as

$$\begin{cases} S_{abc} = \begin{bmatrix} S_a & S_b & S_c \end{bmatrix}^T \\ S_{\alpha\beta 0} = \begin{bmatrix} S_\alpha & S_\beta & S_0 \end{bmatrix}^T \\ S_{dq0} = \begin{bmatrix} S_d & S_q & S_0 \end{bmatrix}^T \end{cases} \quad (2)$$

where S represents the current i, the voltage u or the flux linkage λ, S_0 is the zero-sequence component and others mean the axis component of their index name respectively. The (3) is the rotating transformation that transforms variables from $\alpha\beta$ frame to dq frame.

$$T_{rt} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

As is well known, Clarke transformation[7] alters variables from abc frame to $\alpha\beta$ frame, and it times the rotating transformation (3) equals the Park Transformation (1). If Park transformation is changed to other forms because of some principles such as the principle of power invariance, the Clarke Transformation also needs changing respectively.

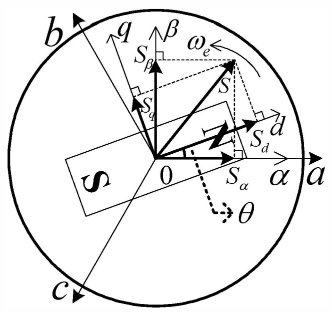

Figure 1 Park transformation

Hence, more generally, we define the extended Clarke Transformation as

$$T_{eck} = k\begin{bmatrix} \cos 0 & \cos(2\pi/3) & \cos(4\pi/3) \\ \sin 0 & \sin(2\pi/3) & \sin(4\pi/3) \\ a & a & a \end{bmatrix} \quad (4)$$

Accordingly, the extended Park (EPark) Transformation can be defined as

$$T_{ep} = k\begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta-4\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta-4\pi/3) \\ a & a & a \end{bmatrix} \quad (5)$$

which can be resolved as three steps of transformation. Firstly, alter variables from abc frame to $\alpha\beta$ fame under the principle of projection. Second, resize the variables of $\alpha\beta$ fame with the scale factor k. Finally, it's the rotating transformation from $\alpha\beta$ fame to dq frame with the angle θ clockwise. The a is the ratio of zero-sequence component. So there are

$$\begin{cases} T_{ep} = T_{rt} T_{eck} \\ S_{\alpha\beta 0} = T_{eck} S_{abc} \\ S_{dq0} = T_{rt} S_{\alpha\beta 0} = T_{ep} S_{abc} \end{cases} \quad (6)$$

where the S stands for the current i, the voltage u or the flux linkage λ. In additions, their inverse operations are also right while T_{ep} and T_{eck} are nonsingular. The inverse T_{ep} is

$$T_{ep}^{-1} = \frac{2}{3k}\begin{bmatrix} \cos\theta & -\sin\theta & 1/(2a) \\ \cos(\theta-2\pi/3) & -\sin(\theta-2\pi/3) & 1/(2a) \\ \cos(\theta-4\pi/3) & -\sin(\theta-4\pi/3) & 1/(2a) \end{bmatrix} \quad (7)$$

where the k and the a can't be zero.

III. GENERALIZED DQ MODEL OF THE PMSM

The stator of the PMSM is similar to the wound rotor SM's. The rotor excitation field of the PMSM is produced by permanent magnets rather than by the coil current in the SM, which cuts the loss of the rotor of the PMSM effectively. Specially, there is no difference between the back EMF by the permanent magnet and that by the coil. So the dq model of the PMSM is similar to the SM's and the generalized dq (GDQ) model of the PMSM can be derived from the SM based on the EPark transformation above. Accordingly, the following assumptions are made in the modeling derivation.

1) Core saturation is neglected.

2) Hysteresis and eddy currents losses are negligible.

3) The EMF is sinusoidal and harmonic is neglected.

4) There is no damper winding on the rotor.

With the assumptions and the EPark transformation above, the GDQ model can be derived fluently as follow.

A. Generalized flux linkage equations

With the EPark transformation, the generalized dq flux linkage equations can be derived from abc flux linkages. According to the definition, flux linkage is related very close to the inductance. The stator abc frame inductances model of the PMSM is similar to the SM's, which is

$$L_{ss} = \begin{bmatrix} L_{aa} & M_{ab} & M_{ac} \\ M_{ba} & L_{bb} & M_{bc} \\ M_{ca} & M_{cb} & L_{cc} \end{bmatrix} \quad (8)$$

where elements of the main diagonal are the self-inductances of their index phases, and others are mutual inductances of their second index phase to first index phase respectively. More than that, the permanent magnet flux linkage (9) of the PMSM can be equivalent to flux linkage produced by virtual windings like the rotor of the SM

$$\lambda_{pm} = L_{af} i_f \quad (9)$$

where L_{af} is the virtual mutual inductance when some stator phase axis points to the d axis of the rotor. And if θ is electric angle between a-phase axis and d axis, virtual mutual inductance vector between stator and rotor can be

$$L_{sr} = [\cos\theta \ \cos(\theta - 2\pi/3) \ \cos(\theta - 4\pi/3)]^T L_{af} \quad (10)$$

Hence, the stator flux linkage vector is

$$\lambda_{abc} = L_{ss} i_{abc} + L_{sr} i_f \quad (11)$$

Premultiply both left part and right part of (11) by (5), plug (10) into (11), and (11) will be simplified to

$$\lambda_{dq0} = T_{ep} L_{ss} T_{ep}^{-1} i_{dq0} + \begin{bmatrix} 3k/2 \\ 0 \\ 0 \end{bmatrix} L_{af} i_f \quad (12)$$

According to the detailed stator inductances model in [8] and the EPark transformation (5), there is

$$T_{ep} L_{ss} T_{ep}^{-1} = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_0 \end{bmatrix} \quad (13)$$

where the calculated results of dq0 inductances are

$$\begin{cases} L_d = L_{a1} + 3(L_{aa0} + L_{g2})/2 \\ L_q = L_{a1} + 3(L_{aa0} - L_{g2})/2 \\ L_0 = L_{a1} \end{cases} \quad (14)$$

In (14), L_{a1} is the armature self-inductance component caused by leakage flux linkage, L_{aa0} is the armature self-inductance component caused by main flux linkage of air-gap, and L_{g2} is the armature self-inductance component caused by the salient pole characteristic. The L_0 is the zero-sequence inductance. Equations (14) show the fact that the dq0 inductances have nothing to do with the factors k and a in the EPark transformation (5). Put (9), (13) and (14) in (12), there will be the dq0 flux linkage vector

$$\lambda_{dq0} = \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_0 \end{bmatrix} i_{dq0} + \begin{bmatrix} 3k/2 \\ 0 \\ 0 \end{bmatrix} \lambda_{pm} \quad (15)$$

which can be shift to the generalized dq flux linkage equations (16) based on the EPark transformation (5).

$$\begin{cases} \lambda_d = L_d i_d + 3k\lambda_{pm}/2 \\ \lambda_q = L_q i_q \end{cases} \quad (16)$$

What is remarkable in (16) is the fact that the d axis flux linkage is related to the factor k in (5).

B. Generalized voltage equations

With KVL and (5), there is the stator voltage vector of abc frame

$$\begin{aligned} u_{abc} &= R_s i_{abc} + p\lambda_{abc} \\ &= R_s T_{ep}^{-1} i_{dq0} + p(T_{ep}^{-1} \lambda_{dq0}) \end{aligned} \quad (17)$$

where R_s is the phase resistance of the stator and p is differential operation. Premultiply both left part and right part of (17) by (5), so (17) will be simplified to

$$u_{dq0} = R_s i_{dq0} + p\lambda_{dq0} + p\theta \cdot \begin{bmatrix} -\lambda_q \\ \lambda_d \\ 0 \end{bmatrix} \quad (18)$$

which can be rewritten to the generalized dq voltage equations (19) based on the EPark transformation (5).

$$\begin{cases} u_d = R_s i_d + p\lambda_d - \omega_e \lambda_q \\ u_q = R_s i_q + p\lambda_q + \omega_e \lambda_d \end{cases} \quad (19)$$

where ω_e is electric angular velocity.

Although it seems that equations (19) are irrelevant to the factor k of (5), there will be (20) if plug (16) into (19)

$$\begin{cases} u_d = R_s i_d + L_d p i_d - \omega_e L_q i_q \\ u_q = R_s i_q + L_q p i_q + \omega_e L_d i_d + 3k\omega_e \lambda_{pm} / 2 \end{cases} \tag{20}$$

which indicates that q axis voltage is relevant to the factor k.

C. Generalized electromagnetic torque equations

Torque is related to power and angular velocity, so it's necessary to obtain the input power equation (21).

$$\begin{aligned} P_{in} &= u_{abc}^T i_{abc} = (T_{ep}^{-1} u_{dq0})^T T_{ep}^{-1} i_{dq0} \\ &= u_{dq0}^T (T_{ep}^{-1})^T T_{ep}^{-1} i_{dq0} \\ &= u_{dq0}^T (T_{ep} T_{ep}^T)^{-1} i_{dq0} \end{aligned} \tag{21}$$

Put (5) in, and the result is

$$P_{in} = 2 \frac{u_d i_d + u_q i_q + u_0 i_0 / (2a^2)}{3k^2} \tag{22}$$

where zero-sequence power component is usually considered as zero. So put (19) in, it will be

$$\begin{aligned} P_{in} = 2(i_d \cdot p\lambda_d + i_q \cdot p\lambda_q) / 3k^2 + \\ 2R_s (i_d^2 + i_q^2) / 3k^2 + 2\omega_e (i_q \lambda_d - i_d \lambda_q) / 3k^2 \end{aligned} \tag{23}$$

In (23), the first part is the changing rate of stored magnetic field energy, the second part is the copper loss of stator windings, and the last part is the output power which becomes mechanical power and produces electromagnetic torque. Define the mechanical angular velocity

$$\omega_r = \omega_e / p_n \tag{24}$$

where p_n is the number of pole pairs. With (24) and the third part of (23), there is the generalized electromagnetic torque equation

$$T_{em} = 2p_n (i_q \lambda_d - i_d \lambda_q) / 3k^2 \tag{25}$$

If put (16) in, it will be the other form

$$T_{em} = 2p_n (L_d - L_q) i_d i_q / 3k^2 + p_n \lambda_{pm} i_q / k \tag{26}$$

In (25) and (26), note that it's the factor k of (5) that is related to the electromagnetic torque of the PMSM directly, for which all kinds of editions of PMSM dq model have different forms of electromagnetic torque equation.

All in all, main components of the GDQ model equations of the PMSM based on EPark transformation is derived step by step above. Nevertheless, beside the generalized flux linkage equations (16), the generalized voltage equations (19) or (20), and the generalized electromagnetic torque equations (25) or (26), the GDQ model of the PMSM must also include the mechanical motion equation (27) [9]

$$J \frac{d\omega_r}{dt} = T_{em} - T_L \tag{27}$$

where J is rotational inertia and T_L is the whole load torque.

IV. Model Compatibility Verfication

As is presented above, the GDQ model of the PMSM is composed of (16), (19) or (20), (25) or (26), and (27). From the derivation process and results above, it can be found that the root reason of difference among all editions of PMSM dq model is different forms of EPark transformation (5), which is depended on the factors k and a. When the factors k and a are replaced with various values (>0), the GDQ model will be instantiated to different editions of PMSM dq model. So the model compatibility verification work should be focused on the equations including the factor k, or a, or both.

Equations (20) are produced by (16) and (19), so are (26) by (16) and (25). Hence, if (16), (19) and (25) are compatible with other editions of PMSM dq model, so will be (20) and (26). In additions, (19) don't include the factor k or a, so the whole model's compatibility hinges on the (16) and (25)'s. For example, the two widely used editions of PMSM dq model are selected to verify their compatibility with the GDQ model as follow. Others can be done similarly.

A. Model edition based on Park transformation

The dq model edition based on Park transformation is the most widely used editions of PMSM dq model. If k=2/3 and a=1/2, the EPark transformation (5) is the same as Park transformation (1), (16) will be (28), and (25) will be (29). So the GDQ model equations are instantiated to (28), (19), (29) and (27), which are almost the same as other dq model editions' equations[4] based on Park transformation with possible difference of variable symbols or machine's running mode (motor or generator) only.

$$\begin{cases} \lambda_d = L_d i_d + \lambda_{pm} \\ \lambda_q = L_q i_q \end{cases} \tag{28}$$

$$T_{em} = 1.5 p_n (i_q \lambda_d - i_d \lambda_q) \tag{29}$$

B. Model edition based on of principle of power invariance

Especially in Chinese literatures, the dq model edition based on the principle of power invariance is also a widely used edition of PMSM dq model. If

$$\begin{cases} k = \sqrt{2}/\sqrt{3} \\ a = 1/\sqrt{2} \end{cases} \tag{30}$$

(5), (16), and (25) will be (31)~(33) respectively.

$$T_{ep} = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta-4\pi/3) \\ -\sin\theta & -\sin(\theta-2\pi/3) & -\sin(\theta-4\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{31}$$

$$\begin{cases} \lambda_d = L_d i_d + \sqrt{3}\lambda_{pm}/\sqrt{2} \\ \lambda_q = L_q i_q \end{cases} \tag{32}$$

$$T_{em} = p_n(i_q\lambda_d - i_d\lambda_q) \tag{33}$$

So the GDQ model equations are instantiated to (32), (19), (33) and (27), which are almost the same as other dq model editions' equations based on the principle of power invariance with possible difference of variable symbols or machine's running mode (motor or generator) only.

Note that the permanent magnet flux linkage in (32) is usually resized as (34) in most literatures[10].

$$\lambda_{pm}' = \sqrt{3}\lambda_{pm}/\sqrt{2} \tag{34}$$

So (32) will be

$$\begin{cases} \lambda_d = L_d i_d + \lambda_{pm}' \\ \lambda_q = L_q i_q \end{cases} \tag{35}$$

However, we still suggest use (32) rather than (35), which can prevent confusion with other editions.

In short, it's verified above that GDQ model of the PMSM is compatible with the dq model edition based on Park transformation and that based on the principle of power invariance. And others can be easy to verify similarly.

V. SIMULATION

In the last three decades, a lot of studies have shown that PMSM drives using the dq model edition based on the Park transformation or that based on the principle of power invariance have high performance, which are also widely applied in industry. So there is no doubt that the PMSM drives based on the GDQ model, which is instantiated to the editions based on Park transformation or that based on the principle of power invariance(traditional editions for short here), have high performance, too.

The performance investigation work of traditional editions can be found in many and many literatures, so it's not necessary to do repeat investigation work of those here. The purpose of simulation here is to investigate the performance of the PMSM drive based on the GDQ model, which is instantiated to another edition. On account of space limitation, we show simulation results only when k=1/3 as follow. Other editions can be done similarly.

The simulation block diagram is shown in Fig. 2. As is shown in it, the vector control based on SVPWM is used, and the PMSM is driven by the three-phase voltage source inverter. By the way, resizing transformation step of (7) the reverse of the EPark transformation is integrated into the SVPWM like the inverse of extended Clarke transformation (4).

The parameters of the PMSM are shown in TABLE I. As is show in it, the motor's rotor has the characteristic of little salient-pole.

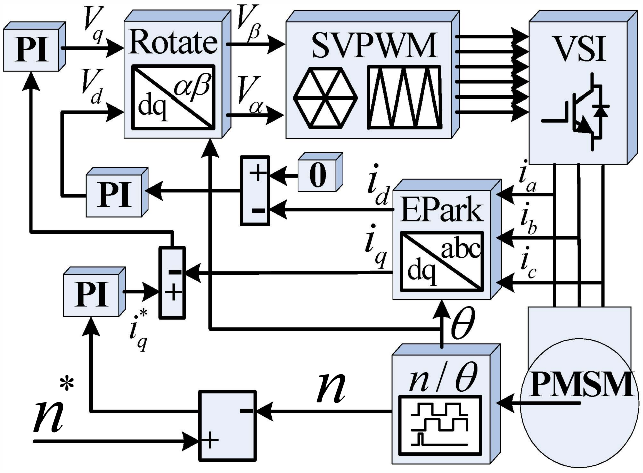

Figure 2 Block diagram for vector controlled PMSM

TABLE I. PARAMETERS OF PMSM

Motor Parameters		
Symbol	***Parameter***	***Value***
R_s	Resistance	0.982 ohm
L_d	d axis inductance	2.9 mH
L_q	q axis inductance	3.0 mH
λ_{pm}	Flux linkage by permanent magnets	0.075 Wb
Vpeak/Krpm	L-L voltage constant	54.167 V/krpm
p_n	The number of pole pairs	4 pairs
J	rotational inertia	0.425x10^{-3} kgm^2

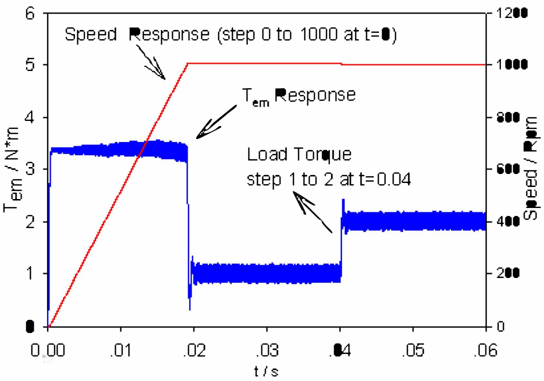

Figure 3 (a) Speed response and torque response

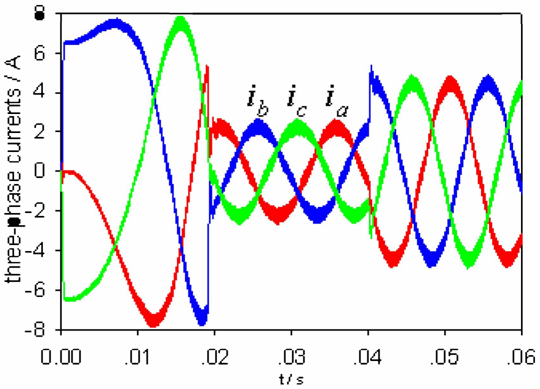

Figure 3 (b) Three-phase currents of the PMSM stator

Fig. 3 (a) shows the speed response and electromagnetic torque response which includes the speed step 0 to 1000 rpm at t=0s and the load torque step 1 to 2 N*m at t=0.04s. Fig. 3 (b) shows the three-phase currents of the PMSM stator.

The results presented here show the satisfactory response performance of the vector controlled PMSM based on the generalized dq model with the EPark transformation's factor k=1/3. This confirms the GDQ model can still operate well when it is instantiated to another edition rather than either one of the traditional editions.

VI. CONCLUSION

In this paper the generalized dq model of the PMSM based on the EPark transformation is presented, whose equations' derivation process is shown step by step. The model's compatibility with the most widely used two editions of dq model based on Park transformation and the principle of power invariance is verified. That with others can be done similarly. The vector controlled PMSM drive with the generalized dq model instantiated to another edition is verified by simulation in Matlab which shows the satisfactory response performance under speed step and load torque step.

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